9.4 Exploring the Cosine Law

Try These

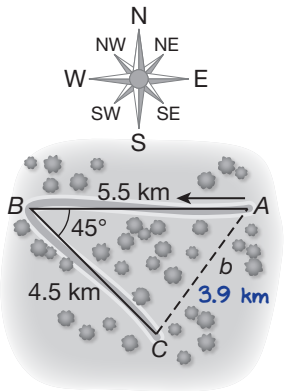
i) $x^2 = 3^2 + 4^2 - (0.5)(14)(3)$

$x =$ 2

ii) $x^2 = 5.3^2 + 2.7^2 - (2)(5.3)(2.7)$

$x =$ 2.6

Tran is a park warden. She walked 5.5 km west from *A* along a straight trail. Then she turned at *B* and walked 4.5 km southeast to *C*. How far was Tran from where she started?



1 Can you use the sine law to solve this problem? Explain.

No. e.g., You would need to know one side and the opposite angle to use the sine law.

cosine law

for $\triangle ABC$, the cosine law is written as $a^2 = b^2 + c^2 - 2bc \cos A$

2 You can use the **cosine law** to solve this problem. Two forms of the cosine law are shown below.

$a^2 = b^2 + c^2 - 2bc \cos A$ $\qquad$ $b^2 = a^2 + c^2 - 2ac \cos B$

Each form uses the cosine of the angle that is opposite the side whose length you want to calculate. Look at the pattern. What is the third form of the cosine law?

$c^2 =$ ___a___ $^2 +$ ___b___ $^2 - 2$ ___ab___ $\cos$ ___C___

Tech Tip

$4^2 + 5^2 - 2(4)(5)\cos 30°$

To calculate in one step, enter

4 [x^2] [+] 5 [x^2] [−] 2 [×] 4 [×] 5 [×] [cos] 30 [=]

3 How far was Tran from point *A*, where she started?

$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 =$ 4.5 $^2 +$ 5.5 $^2 - 2($ 4.5 $)($ 5.5 $)\cos$ 45 °

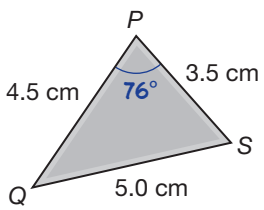
$b^2 =$ 20.25 $+$ 30.25 $-$ 49.5 $\cos$ 45 °

$b^2 =$ 15.498...

$b =$ 3.936... Tran was 3.9 km from where she started.

REFLECTING

Why does it make sense to use the cosine law to calculate b^2?



Example

Tijana makes pendants from stained glass. She sells the pendants at craft shows. She needs a triangular piece of glass with the dimensions shown. At what angle should Tijana cut the glass at *P*? Record the answer on the diagram.

Solution 1: Using the cosine law

A. What are the side lengths?

$p =$ 5.0 cm $\quad q =$ 3.5 cm $\quad s =$ 4.5 cm

B. At what angle should Tijana cut the glass at P?

$$p^2 = q^2 + s^2 - 2qs\cos P$$
$$5.0^2 = 3.5^2 + 4.5^2 - 2(3.5)(4.5)\cos P$$
$$25 = 12.25 + 20.25 - 31.5\cos P$$
$$25 - 12.25 - 20.25 = -31.5\cos P$$
$$-7.5 = -31.5\cos P$$
$$\frac{-7.5}{-31.5} = \cos P$$
$$\cos^{-1}\left(\frac{7.5}{31.5}\right) = \angle P$$
$$76.225...^\circ = \angle P$$

Tijana should cut the glass at an angle of 76 °.

Solution 2: Rearranging the cosine law

A. Substitute the values for the three sides into the rearranged cosine law. Solve for $\angle P$.

$$\cos P = \frac{q^2 + s^2 - p^2}{2qs}$$
$$\cos P = \frac{3.5^2 + 4.5^2 - 5.0^2}{2(3.5)(4.5)}$$
$$\cos P = \frac{7.5}{31.5}$$
$$\angle P = \cos^{-1}\left(\frac{7.5}{31.5}\right)$$

$\angle P =$ 76.225...°, or 76 °

Hint

Rearranging the cosine law

To determine an angle, you can write the cosine law like this:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Tech Tip

Use brackets around both the numerator and the denominator. To calculate $\frac{5^2 + 8^2 - 6^2}{2(5)(8)}$, enter

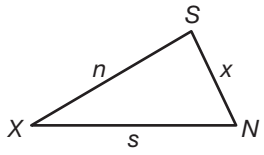
(5 x^2 + 8 x^2 − 6 x^2) ÷ (2 × 5 × 8) =

Practice

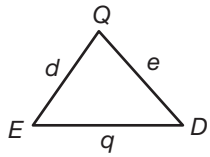
REFLECTING
How can you write the cosine law in Question 1, Part a) to determine $\angle S$?

1. Write the form of the cosine law needed to determine each measure.

a) $\angle N$



b) side ED

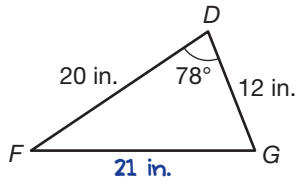


$n^2 = s^2 + x^2 - 2sx \cos N$ OR $\cos N = \dfrac{s^2 + x^2 - n^2}{2sx}$

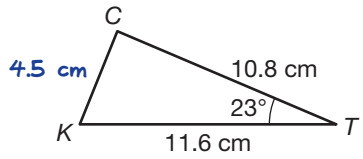
$q^2 = d^2 + e^2 - 2de \cos Q$

2. What is the measure of the unknown side length in each triangle?

a)



b)



$d^2 = f^2 + g^2 - 2fg \cos D$

$d^2 = 12^2 + 20^2 - 2(12)(20) \cos 78°$

$d^2 = 444.202...$

$d = 21.076...$, or 21 in.

e.g., $t^2 = k^2 + c^2 - 2kc \cos T$

$t^2 = 10.8^2 + 11.6^2 - 2(10.8)(11.6) \cos 23°$

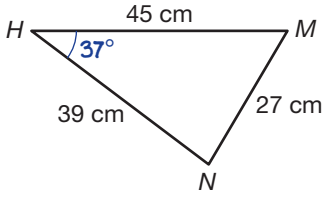
$t^2 = 20.558...$

$t = 4.534...$, or 4.5 cm

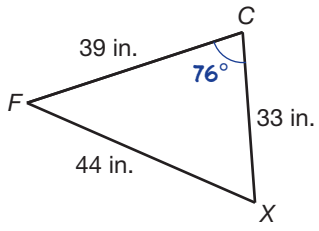
REFLECTING
Which method for determining the angle measure in Question 3 did you prefer? Why?

3. What is the measure of each angle?

a) $\angle H$



b) $\angle C$



$\cos H = \dfrac{m^2 + n^2 - h^2}{2mn}$

$\cos H = \dfrac{39^2 + 45^2 - 27^2}{2(39)(45)}$

$\cos H = \dfrac{2817}{3510}$

$\angle H = \cos^{-1}\left(\dfrac{2817}{3510}\right)$

$\angle H = 36.624...°$, or 37°

$c^2 = f^2 + x^2 - 2fx \cos C$

$44^2 = 33^2 + 39^2 - 2(33)(39) \cos C$

$1936 = 1089 + 1521 - 2754 \cos C$

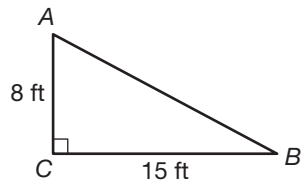
$1936 - 1089 - 1521 = -2754 \cos C$

$\dfrac{-674}{-2754} = \cos C$

$\cos^{-1}\left(\dfrac{-674}{-2754}\right) = \angle C$

$75.833...° = \angle C$, or 76°

4. a) Determine the length of side c using each method.



Cosine law	Pythagorean theorem
$c^2 = a^2 + b^2 - 2ab \cos C$	$c^2 = a^2 + b^2$
$c^2 = 15^2 + 8^2 - 2(15)(8) \cos 90°$	$c^2 = 15^2 + 8^2$
$c^2 = 289$	$c^2 = 289$
$c = 17$, or 17 ft	$c = 17$, or 17 ft

b) What is the value of $2ab \cos C$ if $\angle C$ is a right angle? 0

c) Simplify the cosine law if $\angle C$ is a right angle.

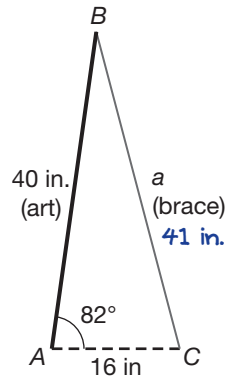
$c^2 = a^2 + b^2 - 2ab \cos 90°$

What is the result?

e.g., The cosine law becomes $c^2 = a^2 + b^2$.

This is the Pythagorean theorem.

5. Dominique is a window dresser for a department store. This diagram shows how she plans to use a brace to support a collage that is 40 in. tall. How long does the brace need to be?



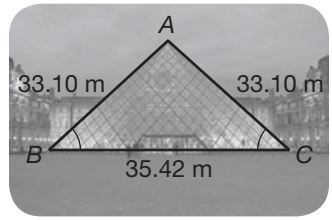
e.g., $a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 16^2 + 40^2 - 2(16)(40) \cos 82°$

$a^2 = 1677.858...$

$a = 40.961...$ The brace needs to be 41 in. long.

6. The Louvre Pyramid, in Paris, is at the main entrance to the Louvre Museum. Its dimensions are shown on the photo at the right. What is the measure of each angle between the base and the side edges?



e.g., $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \dfrac{35.42^2 + 33.10^2 - 33.10^2}{2(35.42)(33.10)}$

$\angle B = \cos^{-1}\left(\dfrac{1254.5764}{2344.804}\right)$, or 57.653...

Each angle between the base and the side edges is 58°.

7. Jaani is a contractor in Hanna. How might Jaani use the cosine law when he is building a roof?

e.g., If Jaani knows the length of each rafter and the width of the structure, he can use the cosine law to calculate the angles.