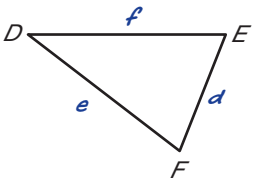
9.2 Solving Sine-Law Problems

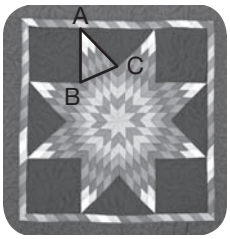
Try These

Label the sides of the triangle. Complete the sine law for $\triangle DEF$.

i) $\dfrac{d}{\sin \boxed{D}} = \dfrac{\boxed{e}}{\sin E} = \dfrac{\boxed{f}}{\sin \boxed{F}}$

ii) $\dfrac{\sin D}{\boxed{d}} = \dfrac{\sin \boxed{E}}{\boxed{e}} = \dfrac{\sin \boxed{F}}{f}$





The eight-pointed morning-star quilt is a traditional gift in Sioux culture. The points of the star are congruent isosceles triangles. For $\triangle ABC$, base BC is 21.5 in. and $\angle A$ is 46°. What is the length of sides AB and CA?

REFLECTING

Why was it necessary to determine the measure of $\angle B$? Could you have used the measure of $\angle C$ instead?

1. What is the measure of $\angle B$?

$2(\angle B) = 180° - \boxed{46°}$

$2(\angle B) = \boxed{134°}$

$\angle B = \boxed{67°}$

2. What is the length of sides AB and CA?

$\dfrac{b}{\sin \boxed{67}°} = \dfrac{\boxed{21.5}}{\sin \boxed{46}°}$

$b = \sin \boxed{67}° \left(\dfrac{\boxed{21.5}}{\sin \boxed{46}°} \right)$

$b = \boxed{27.512...}$

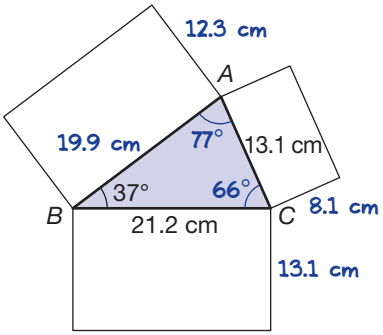
The length of sides AB and CA is $\boxed{27.5}$ in.

golden rectangle
a rectangle with a length that is about 1.618 times its width; the golden rectangle is often used in art and in architecture

Example

Grant is making a stained-glass window. He wants to use the triangle in the diagram at the right. Each side of the triangle will be the base of a **golden rectangle**.

What will be the dimensions of each golden rectangle?



Solution

A. What is the measure of $\angle A$?

Determine $\sin A$.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{\boxed{21.2}} = \frac{\sin \boxed{37}^\circ}{\boxed{13.1}}$$

$$\sin A = \underline{21.2}\left(\frac{\sin \boxed{37}^\circ}{\boxed{13.1}}\right)$$

$$\sin A = \underline{0.9739\ldots}$$

Determine $\angle A$.

$\angle A = \sin^{-1}(\underline{0.9739\ldots})$

$\angle A = \underline{76.888\ldots}^\circ$

Hint

Keep all decimal digits in Parts A and B for later calculations.

B. What is the measure of $\angle C$? What is the length of side c?

$\angle C = \underline{180}^\circ - \angle A - \angle B$

$\angle C = \underline{180}^\circ - \underline{76.888\ldots}^\circ - \underline{37}^\circ$, or $\underline{66.111\ldots}^\circ$

$$\frac{c}{\sin \boxed{66.111\ldots}^\circ} = \frac{\boxed{13.1}}{\sin \boxed{37}^\circ}$$

$$c = \sin \underline{66.111\ldots}^\circ \left(\frac{\boxed{13.1}}{\sin \boxed{37}^\circ}\right), \text{ or } \underline{19.902\ldots}$$

REFLECTING

Why do you think two versions of the sine law are used for Parts A and B?

C. What will be the dimensions of each golden rectangle?

Golden rectangle on AC:

Length: 13.1 cm

Width: 13.1 cm ÷ 1.618 = $\underline{8.096\ldots}$ cm

Golden rectangle on BC:

Length: 21.2 cm

Width: 21.2 cm ÷ 1.618 = $\underline{13.102\ldots}$ cm

Golden rectangle on AB:

Length: $\underline{19.902\ldots}$ cm

Width: $\underline{19.902\ldots}$ cm ÷ 1.618 = $\underline{12.300\ldots}$ cm

The rectangle on AC is 13.1 cm by $\underline{8.1}$ cm.

The rectangle on BC is 21.2 cm by $\underline{13.1}$ cm.

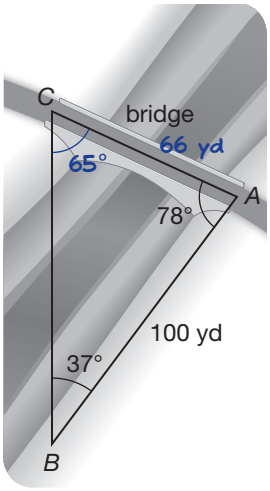
The rectangle on AB is $\underline{19.9}$ cm by $\underline{12.3}$ cm.

The construction of the Acropolis in Greece uses the golden rectangle.

Hint

Record calculated answer measurements on the diagrams.

Practice



1. Lucien is building a bridge across a creek in British Columbia. He took measurements along shoreline *AB*, as shown. What is the length of the bridge?

e.g., $\angle C = 180° - \angle A - \angle B$

$\angle C = 180° - 37° - 78°$

$\angle C = 65°$

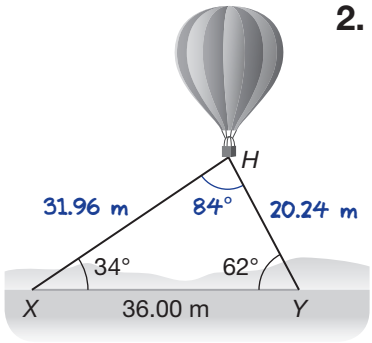
e.g., $\frac{b}{\sin B} = \frac{c}{\sin C}$

$\frac{b}{\sin 37°} = \frac{100}{\sin 65°}$

$b = \sin 37°\left(\frac{100}{\sin 65°}\right)$

$b = 66.402...$

The bridge is 66 yd long.



2. Anya used cables to anchor an advertising balloon above a building. The cables are attached to the ground, 36.00 m apart, at *X* and *Y*. The angles of elevation to the bottom of the balloon are 34° and 62°.

How long are the cables? Do not include the length needed to secure them to the ground.

$\angle H = 180° - 34° - 62°$

$\angle H = 84°$

Cable *XH*:

$\frac{y}{\sin 62°} = \frac{36}{\sin 84°}$

$y = \sin 62°\left(\frac{36}{\sin 84°}\right)$

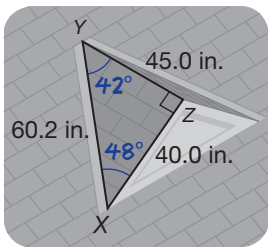
$y = 31.961...$, or 31.96 m

Cable *YH*:

$\frac{x}{\sin 34°} = \frac{36}{\sin 84°}$

$x = \sin 34°\left(\frac{36}{\sin 84°}\right)$

$x = 20.241...$, or 20.24 m



3. Madison is a roofer in Manitoba. She is shingling dormers on a roof, as shown at the right. $\angle YZX$ is 90°. At what angles along *YX* must Madison cut the shingles on the dormer?

e.g., $\frac{\sin Y}{40} = \frac{\sin 90°}{60.2}$

$\sin Y = 40\left(\frac{\sin 90°}{60.2}\right)$

$\sin Y = 0.6644...$

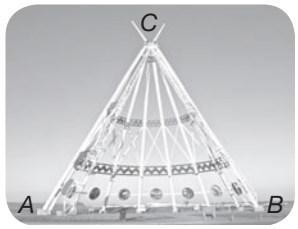
$\angle Y = \sin^{-1}(0.6644...)$

$\angle Y = 41.640...°$

If Madison cuts down from $\angle Y$, she should cut at an angle of 42°. If she cuts up from $\angle X$, she should cut at an angle of 48°.

4. The Saamis Teepee was built for the 1988 Calgary Olympics as a symbol of Canada's Aboriginal heritage. It was moved to Medicine Hat in 1991. How long is each pole?

- The diameter of the base is 160 ft.
- The poles that form the teepee are at an angle of 68° with the ground.



e.g. $\angle C = 180° - \angle A - \angle B$

$\angle C = 180° - 68° - 68°$

$\angle C = 44°$

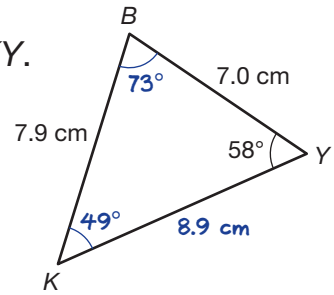
$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 68°} = \frac{160}{\sin 44°}$$

$$a = \sin 68°\left(\frac{160}{\sin 44°}\right)$$

$a = 213.557...$ Each pole is 214 ft long.

5. A triangle is solved when all three side lengths and all three angle measures are known. Follow these steps to solve $\triangle BKY$.



a) Determine $\angle K$.

e.g., $$\frac{\sin K}{k} = \frac{\sin Y}{y}$$

$$\frac{\sin K}{7.0} = \frac{\sin 58°}{7.9}$$

$$\sin K = 7.0\left(\frac{\sin 58°}{7.9}\right)$$

$\sin K = 0.7514...$

$\angle K = \sin^{-1}(0.7514...)$

$\angle K = 48.714...°$, or $49°$

b) Determine $\angle B$.

e.g., $\angle B = 180° - 48.714...° - 58°$

$\angle B = 73.285...°$, or $73°$

c) Determine side b.

e.g., $$\frac{b}{\sin B} = \frac{y}{\sin Y}$$

$$\frac{b}{\sin 73.285...°} = \frac{7.9}{\sin 58°}$$

$$b = \sin 73.285...°\left(\frac{7.9}{\sin 58°}\right)$$

$b = 8.921...$, or 8.9 cm