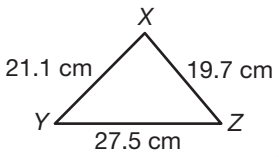
9.1 Exploring the Sine Law

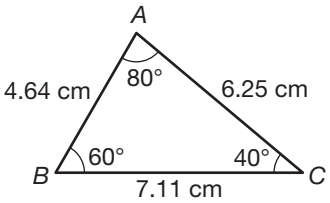
Try These

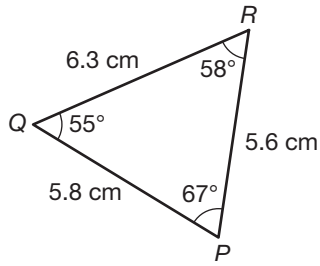
Record the angles in $\triangle XYZ$ from least to greatest.

$\angle$ Y, $\angle$ Z, $\angle$ X



Janelle is a commercial artist in Fort McMurray. Two of her triangular logo designs are shown below.





What is the relationship between the sides and the angles that are opposite the sides?

1 Use $\triangle ABC$. What is the value of each ratio?

$\frac{a}{\sin A} = \frac{7.11}{\sin 80°}$ $= 7.219...$

$\frac{b}{\sin B} = \frac{6.25}{\sin 60°}$ $= 7.216...$

$\frac{c}{\sin C} = \frac{4.64}{\sin 40°}$ $= 7.218...$

REFLECTING

How do your answers for Questions 2 and 4 show the **sine law**?

2 Compare the ratios in Question 1. What do you notice?

The ratios are e.g., all very close to 7.2. They are almost equal.

3 Use $\triangle PQR$. What is the value of each ratio?

$\frac{p}{\sin P} = \frac{6.3}{\sin 67°}$ $= 6.844...$

$\frac{q}{\sin Q} = \frac{5.6}{\sin 55°}$ $= 6.836...$

$\frac{r}{\sin R} = \frac{5.8}{\sin 58°}$ $= 6.839...$

sine law

for $\triangle ABC$, the sine law is written as

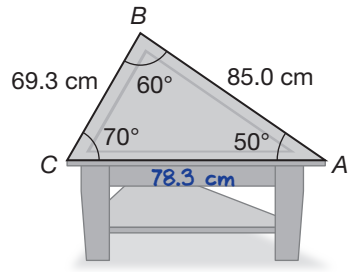
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

4 Compare the ratios in Question 3. What do you notice?

The ratios are e.g., all very close to 6.8. They are almost equal.

Example 1

Logan builds custom tables. A client wants a table with the dimensions at the right. How long is the third side?



Solution

A. What are the known values?

$\angle A =$ 50 ° $\quad \angle B =$ 60 ° $\quad \angle C =$ 70 °

$a =$ 69.3 cm $\quad b = ?$ $\quad c =$ 85.0 cm

Hint

Use the given measurements to determine the precision for answers.

B. How can you use the sine law to calculate the length of side b?

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 60°} = \frac{85.0}{\sin 70°}$$

$$\sin 60°\left(\frac{b}{\sin 60°}\right) = \sin 60°\left(\frac{85.0}{\sin 70°}\right)$$

$$b = \sin 60°\left(\frac{85.0}{\sin 70°}\right), \text{ or } 78.336...$$

The third side is 78.3 cm.

REFLECTING

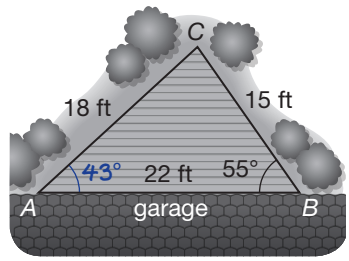
Logan used these ratios to determine the length of side b in Example 1:

$$\frac{b}{\sin B} = \frac{a}{\sin A}$$

Should his answer be the same as your answer? Explain.

Example 2

Alaina is building a triangular deck in Prince Rupert. At what angle to the garage should Alaina place the beam for side AC?



Solution

A. At what angle, $\angle A$, should the beam be?

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{15} = \frac{\sin 55°}{18}$$

$$\sin A = 15\left(\frac{\sin 55°}{18}\right)$$

$$\sin A = 0.6826...$$

$$\angle A = \sin^{-1}(0.6826...), \text{ or } 43.049...°$$

Alaina should place the beam at an angle of 43 °.

Hint

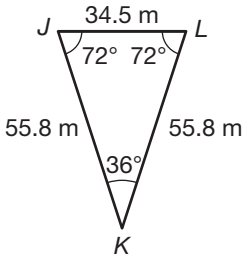
To calculate an angle, you can write the sine law this way:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

REFLECTING

In Example 2, why was it helpful to write the sine law with the angles on top?

Practice



1. What is the value of each ratio, to one decimal place?

$\frac{j}{\sin J} = \frac{\boxed{55.8}}{\sin \boxed{72}^\circ}$ $\quad \frac{k}{\sin K} = \frac{\boxed{34.5}}{\sin \boxed{36}^\circ}$ $\quad \frac{l}{\sin L} = \frac{\boxed{55.8}}{\sin \boxed{72}^\circ}$

$\frac{j}{\sin J} =$ 58.671..., $\quad \frac{k}{\sin K} =$ 58.694..., $\quad \frac{l}{\sin L} =$ 58.671...,

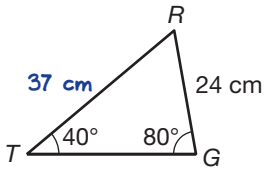
or 58.7 $\quad$ or 58.7 $\quad$ or 58.7

REFLECTING

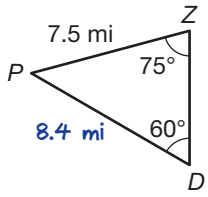
How do you know that the ratios $\frac{j}{\sin J}$ and $\frac{l}{\sin L}$ are equal, without calculating them?

2. Calculate each side length.

a) side g



b) side z



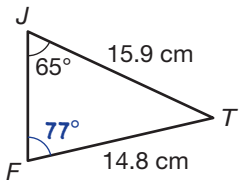
$\frac{g}{\sin \boxed{80}^\circ} = \frac{\boxed{24}}{\sin \boxed{40}^\circ}$ $\quad \frac{z}{\boxed{\sin 75^\circ}} = \frac{\boxed{7.5}}{\boxed{\sin 60^\circ}}$

$g = \sin$ 80 $^\circ\left(\frac{\boxed{24}}{\sin \boxed{40}^\circ}\right)$ $\quad z =$ $\boxed{\sin 75^\circ}\left(\frac{\boxed{7.5}}{\boxed{\sin 60^\circ}}\right)$

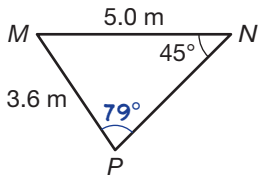
$g =$ 36.770..., or 37 cm $\quad z =$ 8.365..., or 8.4 mi

3. Calculate each angle measure.

a) $\angle F$



b) $\angle P$



$\frac{\sin F}{\boxed{15.9}} = \frac{\sin \boxed{65}^\circ}{\boxed{14.8}}$ $\quad \frac{\sin P}{\boxed{5.0}} = \frac{\boxed{\sin 45^\circ}}{\boxed{3.6}}$

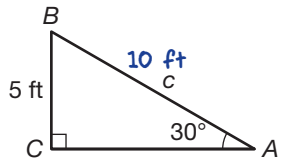
$\sin F =$ 15.9 $\left(\frac{\boxed{\sin 65^\circ}}{\boxed{14.8}}\right)$ $\quad \sin P =$ 5.0 $\left(\frac{\boxed{\sin 45^\circ}}{\boxed{3.6}}\right)$

$\sin F =$ 0.9736... $\quad \sin P =$ 0.9820...

$\angle F = \sin^{-1}($ 0.9736... $)$ $\quad \angle P = \sin^{-1}($ 0.9820... $)$

$\angle F =$ 76.822...°, or 77 ° $\quad \angle P =$ 79.140...°, or 79 °

4. a) Determine the length of side c for the triangle at the right. Use each method.



Sine ratio	Sine law
$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$	$\frac{c}{\sin C} = \frac{a}{\sin A}$
$\sin 30° = \frac{5}{c}$	$\frac{c}{\sin 90°} = \frac{5}{\sin 30°}$
$c(\sin 30°) = 5$	$c = \sin 90°\left(\frac{5}{\sin 30°}\right)$
$c = \frac{5}{\sin 30°}$	$c = 10$, or 10 ft
$c = 10$, or 10 ft	

b) Why do both methods in Part a) have the same result?

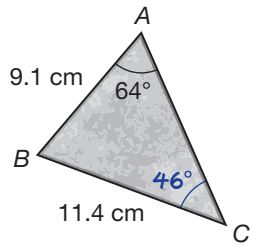
e.g., Since $\sin 90° = 1$, $\sin 90°\left(\frac{5}{\sin 30°}\right) = \frac{5}{\sin 30°}$.

Hint

In $\triangle ABC$, C is 90°. What is the value of sin 90°?

c) Is the sine law true for right triangles? yes

5. Jace creates mosaics using ceramic tiles. He needs to cut a tile as shown at the right. What is the measure of $\angle C$?



e.g., $\frac{\sin C}{c} = \frac{\sin A}{a}$

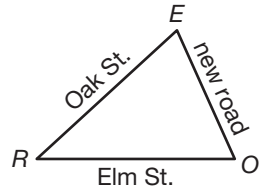
$\frac{\sin C}{9.1} = \frac{\sin 64°}{11.4}$

$\sin C = 9.1\left(\frac{\sin 64°}{11.4}\right)$

$\sin C = 0.7174...$

$\angle C = 45.845...°$, or 46°

6. Amy is a city planner in Victoria. She plans new roads. Show how Amy can use the sine law to calculate the length of the new road in the diagram at the right.



What information does Amy need?

e.g., If Amy knows $\angle O$, $\angle R$, and the length of Oak St., she can use the sine law to figure out the length of the new road from Oak St. to Elm St. $\frac{\text{Oak}}{\sin O} = \frac{\text{new road}}{\sin R}$